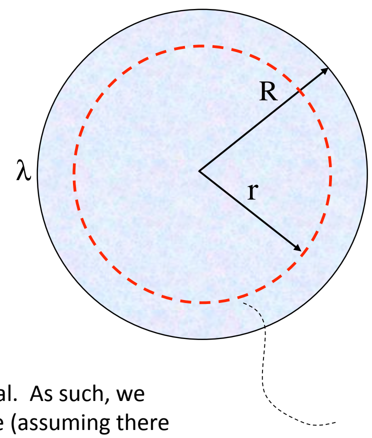
Problem 24.35

Consider a metal rod of radius R = .05 meters with a *charge per unit length* of $\lambda = 30\text{x}10^{-9}\,\text{N/C}$. Derive:

a.) E(r) for r < R:

If this was a non-conducting insulator, it is possible that λ would be telling us how much *charge per unit length* was shot uniformly throughout the cylinder. In that case, would have to determine the "charge enclosed" inside the Gaussian surface to use Gauss's Law. Fortunately, this is a metal. As such, we know all the charge will flow to the outside (assuming there isn't charge stuck inside the structure, which there isn't), so there will be no "charge enclosed" and, hence, E must be ZERO.



b.) E(r) for r > R

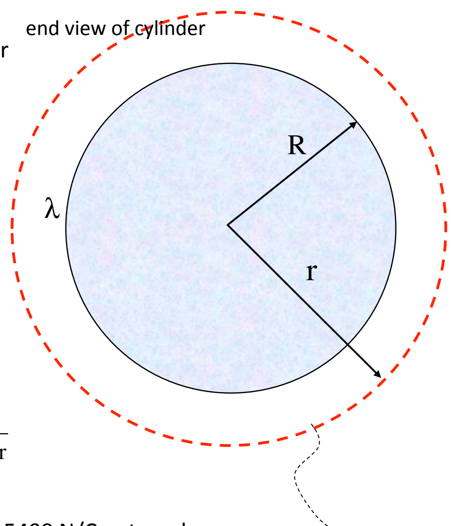
As usual, the left side of Gauss's Law for cylindrical symmetry will be $E(2\pi rL)$, where "L" is the arbitrary length of the Gaussian surface. With that, we can write:

$$\int_A \vec{E}\bullet d\vec{A} = \frac{q_{enclose}}{\varepsilon_o}$$

$$\Rightarrow \quad E(2\pi RL) = \frac{\lambda L}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_o r}$$

$$\Rightarrow \quad E = \frac{(30\text{x}10^{-9}\,C/m)}{2\pi(8.85\text{x}10^{-12}\ C^2/m^2\bullet N)r}$$



b.) doing the math, for r = .1 meters, E = 5400 N/C outward;

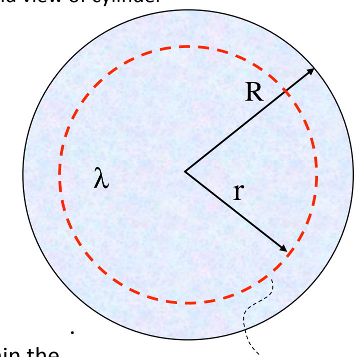
c.) for r = 1 meter, E = 540 N/C outward.

Add-on note:

If the rod had been an insulating non-conductor, then the charge could have been shot though the cylinder (most probably . . . though it could have been put on the surface—for amusement, let's assume it was shot through). In that case:

a.) E for r < R: :

Again, the left side of Gauss's Law will be $E(2\pi rL)$. As not all of the charge will be involved within the Gaussian cylinder (i.e., *r* is smaller than the radius *R* of the actual rod), we need to determine the fraction of charge inside our Gaussian surface. That fraction will be the ratio of the cross-sectional area of the Gaussian end-cap to the cross-sectional area of the whole, or:



$$q_{enclosed} = \left(\frac{A_{inside}}{A_{whole}}\lambda\right)L = \frac{\pi r^2}{\pi R^2}\lambda L = \frac{r^2}{R^2}\lambda L$$

With that, Gauss's Law yields:

$$\int_A \vec{E}\bullet d\vec{A} = \frac{q_{enclose}}{\varepsilon_o}$$

$$\Rightarrow \quad E(2\pi r L) = \frac{\lambda\left(r^2/R^2\right)L}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\lambda r}{2\pi\varepsilon_o R^2}$$

$$\Rightarrow \quad E = \frac{(30\text{x}10^{-9}\,C/m)(.03m)}{2\pi(8.85\text{x}10^{-12}\ C^2/m^2\bullet N)(.05m)^2}$$

$$\Rightarrow \quad E = 6.47\text{x}10^3\ N/C$$

So much for inside the wire. Outside the wire, the E-field would look the same as though all the charge was distributed on the outside of the cylinder (as was the case with the conductor).